(4)

AMSAA

TECHNICAL REPORT NO. 453

STATISTICAL PRECISION AND ROBUSTNESS OF THE AMSAA CONTINUOUS RELIABILITY GROWTH ESTIMATORS

TARIQ ZIAD

PAUL ELLNER

APRIL 1988

U. S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MARYLAND

DISPOSITION

When this report is no longer needed, Department of the Army organizations will destroy it in accordance with the procedures given in AR 380-5. Navy and Air Force elements will destroy it in accordance with applicable directives. Department of Defense contractors will destroy the report according to the requirements of Section 14 of the Industrial Security Manual for Safeguarding Classified Information. All others will return the report to the US Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, Maryland 21005-5071.

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so specified by other official documentation.

WARNING

Information and data contained in this document are based on the input available at the time of preparation. The results may be subject to change and should not be construed as representing the AMC position unless so specified.

TRADE NAMES

The use of trade names in this report does not constitute an official endorsement or approval of the use of such commercial hardware or software. The report may not be cited for purposes of advertisement.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified - TR 453	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION U.S. Army Materiel Systems Analysis Activity 6b. OFFICE SYMBOL (If applicable) AMXSY-RV	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) APG, MD 21005-5071	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION USAMSAA/RAMD 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Statistical Precision and Robustness of the AMSAA Continuous Reliability Growth Estimator

12. PERSONAL AUTHOR(S)
Tariq Ziad and Paul Ellner

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM ______ TO ______	April 28, 1988	

16. SUPPLEMENTARY NOTATION

(Army Materiel Systems Analysis Activity)

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Reliability Growth Model. Precision of (MIL-HDBK-189) MTBF estimators. Robustness of (MIL-HDBK-189) MTBF estimators.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The precision and robustness study of the AMSAA continuous reliability growth estimation procedure investigates the relative error distribution of the MTBF estimator. The study also addresses how this distribution is influenced when the failure data are generated from a finite number of configurations rather than from the AMSAA continuous failure rate curve.

This methodology can be used to calculate the required test time associated with an idealized planning curve to achieve a specified precision with a given probability.

(Mean Time Between Failures)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ XX UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code) / 22c. OFFICE SYMBOL

DD Form 1473, JUN 86 — Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

ACKNOWLEDGEMENTS

The U.S. Army Materiel Systems Analysis Activity (AMSAA) recognizes the following individual for contributing to this report.

Peer Reviewer: James J. Streilein

The authors wish to recognize the following for his contribution to Appendix A.

Robert Mioduski, Reliability, Availability, and Maintainability Division (RAMD)

The next page is blank.

CONTENTS

LIST OF FIGURES

STATISTICAL PRECISION AND ROBUSTNESS OF THE AMSAA CONTINUOUS RELIABILITY GROWTH ESTIMATORS

1. INTRODUCTION

1.1 Discussion of Reliability Growth.

The U.S. Army Materiel Systems Analysis Activity (AMSAA) employs the Weibull process to model reliability growth during a development test phase. Development test programs are generally conducted on a phase by phase basis. The AMSAA reliability growth model is designed for tracking the reliability within a test phase. This model evaluates the reliability growth that results from the introduction of design fixes into the system during test.

Figure 1 illustrates a typical pattern of growth on a phase by phase basis.

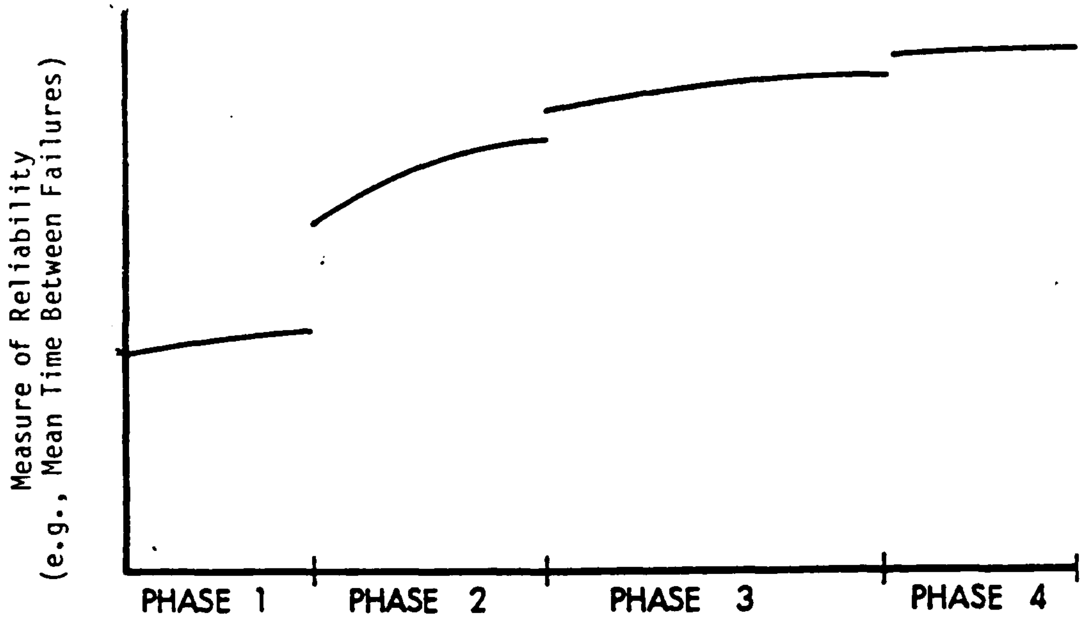


Figure 1. Measure of Reliability (e.g., Mean-Time-Between-Failures (MTBF) in Different Phases.

The AMSAA tracking model addresses the reliability growth within a particular test phase. Several tracking growth curves may be required to measure reliability growth over multiple test phases due to the incorporation of groups of fixes between test phases and/or changes in test phase environments.

Assume the test phase starts at time $t = o$. Within the test phase, let $o<t_1<t_2<\ldots<t_k$ denote the cumulative test times on the system when design modifications are made (see Figure 2).

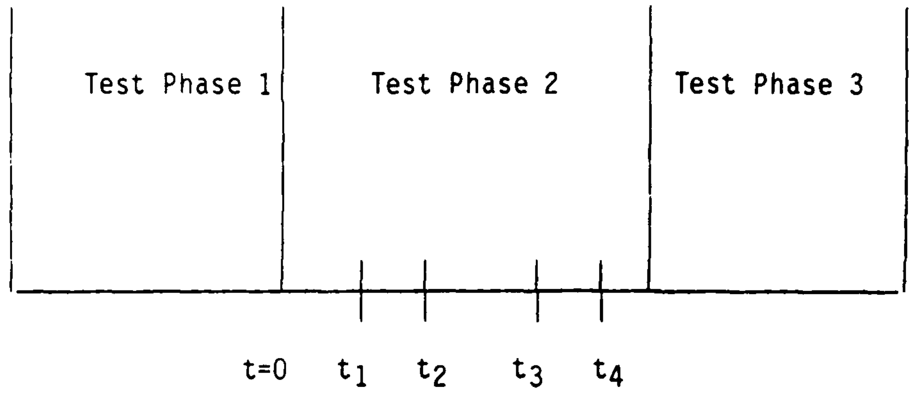


Figure 2. Times of Design Modifications for Test Phase 2.

The failure rate can generally be assumed to be constant between the times when design changes are made on the system. Let λ_i denote the constant failure rate during the ith time period $[t_{i-1}, t_i]$ between modifications (see Figure 3).

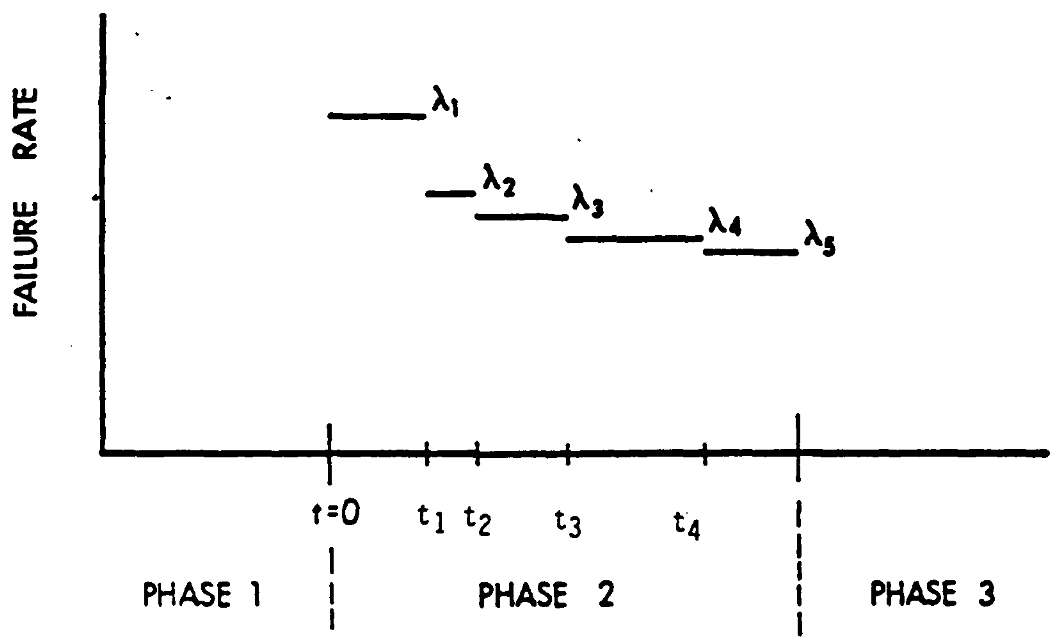


Figure 3. Failure Rates Between Modifications.

The constant failure rate assumption during $[t_{i-1}, t_i]$ implies that for this interval, the times between successive failures follow the exponential distribution $F(x) = 1 - \exp(-\lambda_i x)$, $x>0$.

The AMSAA tracking model approximates the step-wise failure rate function shown in Figure 3 by a smooth curve. The parameters of this curve are estimated, based upon the failure data observed during the test phase.

1.2 Objectives of Our Study.

The objectives of our study are to:

a. Study the statistical precision of the AMSAA MIL-HDBK-189 (Reference 1) mean time between failure (MTBF) estimators M-HAT and M-BAR.

b. Study robustness, i.e., the effect on estimator statistical precision due to discrete configuration changes (i.e., the step-wise discontinuous faibure rate curve).

2. ESTIMATION PROCEDURES ANALYZED

In the subsequent sections, we shall define several important statistical properties of reliability estimators. We shall then study these properties for the MIL-HDBK-189 reliability growth estimation procedures applied to time-terminated testing.

The data used for the analysis consists of the N successive failure times $f_1 < f_2 < f_3 < \ldots < f_N$ which occur during a test phase of duration T. The method of maximum likelihood utilized in the MIL-HDBK-189 estimation procedure provides the estimate of the shape parameter β as

$$\hat{\beta} = \frac{N}{N \ln T - \sum_{i=1}^{N} \ln f_i}$$

where $\ln$ denotes the natural logarithm function.

Subsequently, the scale parameter λ is estimated by $\hat{\lambda} = N/T^{\hat{\beta}}$. It follows that for any time t, the intensity function (failure rate) is estimated by $\hat{\rho}(t) = \hat{\lambda}\hat{\beta}\, t^{\hat{\beta}-1}$. In particular, this holds for T, the total test time. The reciprocal of $\hat{\rho}(T)$ provides an estimate of the MTBF which could be anticipated if the system configuration remains as it is at time T. This estimate is denoted by $\hat{M}$. Thus $\hat{M} = 1/\hat{\rho}(T) = T/N\hat{\beta}$. For small sample sizes, it is appropriate to use an unbiased estimator β of the shape parameter β. The estimator $\bar{\beta}$ is defined in MIL-HDBK-189 as

$$\bar{\beta} = \left(\frac{N-1}{N}\right) \hat{\beta}, \quad N > 2.$$

In our study, we defined the estimator $\bar{\beta}$ as follows:

$$\bar{\beta} = \begin{cases} \hat{\beta} & \text{for } N = 1 \\ \left(\frac{N-1}{N}\right) \hat{\beta} & \text{for } N > 2 \end{cases}$$

Notice the estimator $\bar{\beta}$ defined above is unbiased only for N > 2.

The estimator $\bar{M}$ of MTBF can be calculated by using the unbiased estimator $\bar{\beta}$ as follows:

$$\bar{M} = T/N\bar{\beta} \text{ for } N > 1$$

3. PRECISION

Definition: Precision of an estimator is measured by the Relative Error, RE, defined by

$$RE = |M_{(EST)} - M_{(TRUE)}|/M_{(TRUE)}$$

where $M_{(EST)} = \hat{M}$, calculated from the maximum likelihood estimator $\hat{\beta}$ of the shape parameter β or $M_{(EST)} = \bar{M}$, calculated from the unbiased estimator $\bar{\beta}$ of the shape parameter β. In the above, $M_{(TRUE)}$ denotes the MTBF at the end of the test time.

Since $M_{(EST)}$ is a random variable, RE is a random variable and so we can consider its distribution. This distribution can be simulated by using data generated from the AMSAA continuous failure-rate curve.

It has been found that the probability of achieving a specified precision (i.e., specified relative error) depends solely upon the expected number of failures (see Appendix A). In fact, an analytical expression in terms of the expected number of failures can be found for the distribution function of the relative error. However, for our purposes, we found it more convenient and adequate to use simulation to estimate the probability that the relative error would be less than or equal to a specified value.

In our study we simulated 5,000 failure histories for estimating the probability of achieving a specified precision with $\hat{M}$ and $\bar{M}$. The estimated probabilities were conditioned on the set of failure histories that had at least one failure. For each simulation run, the number of failures N and cumulative failure times $f_1, f_2, f_3, \ldots, f_N$ were recorded. The total test time T was chosen to be 1,000 hours, 5,000 hours, and 10,000 hours. For each test length, the MTBF estimators, $\hat{M}$ and $\bar{M}$, were calculated and, thereby, the distribution of relative error was obtained to analyze the behaviour of the MTBF estimators. It was found that $\hat{M}$ and $\bar{M}$ behave in the same way, especially when the expected number of failures is moderate to large (see Figures 4 and 5).

4. ROBUSTNESS

Definition: Robustness is defined to be the ability of an estimator to perform well even when the underlying assumptions are violated.

For the purpose of our study, we considered a class of step functions which were compatible with the AMSAA tracking model and whose discreteness could be simply characterized (see Appendix C). In the step function construction, the steps represent the constant failure rates over different configurations. By simulation the failure data were generated from the step function failure

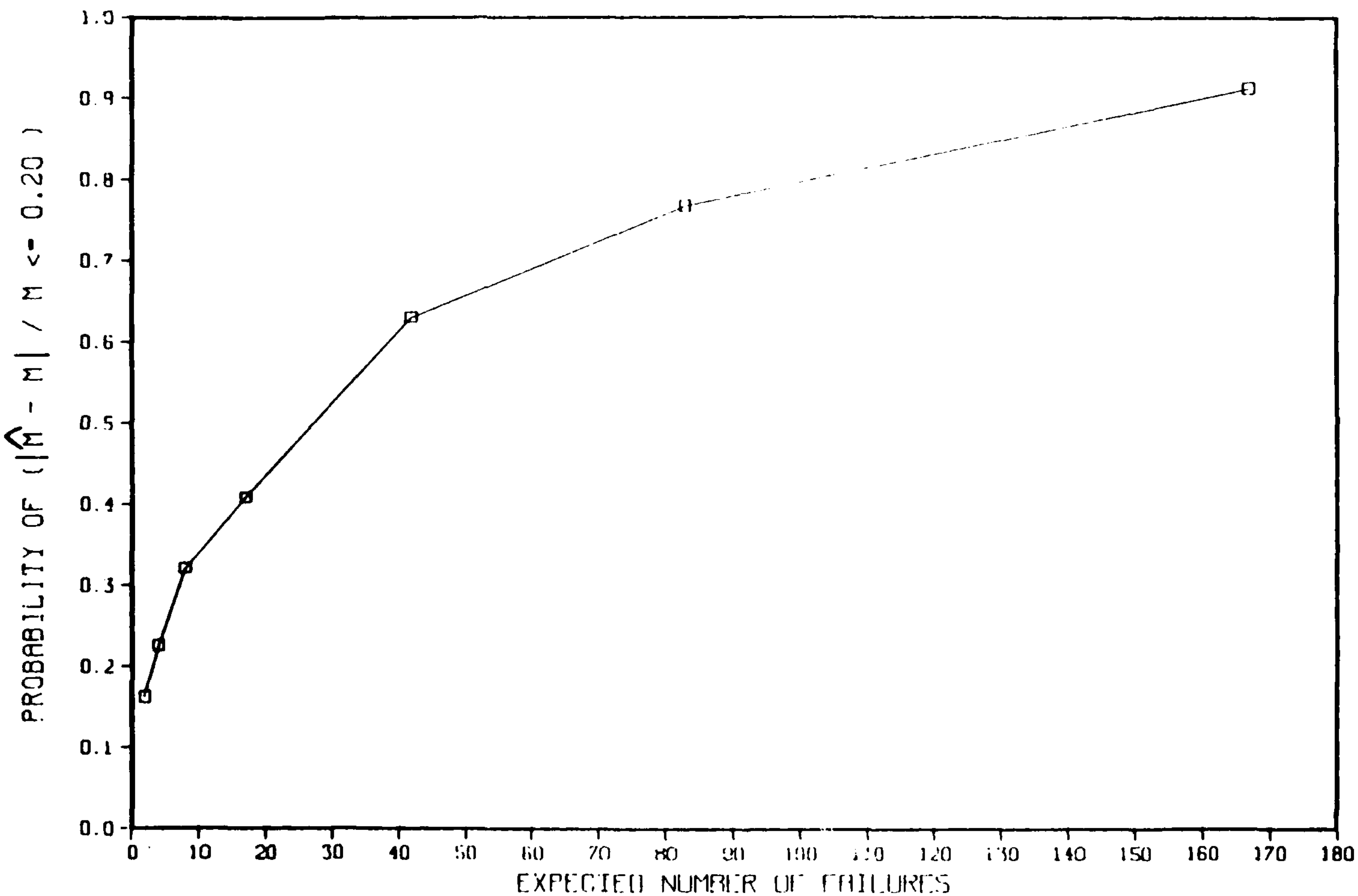


Figure 4. <u>Probability of 20% Precision vs. Expected Number of Failures for $\hat{M}$</u>

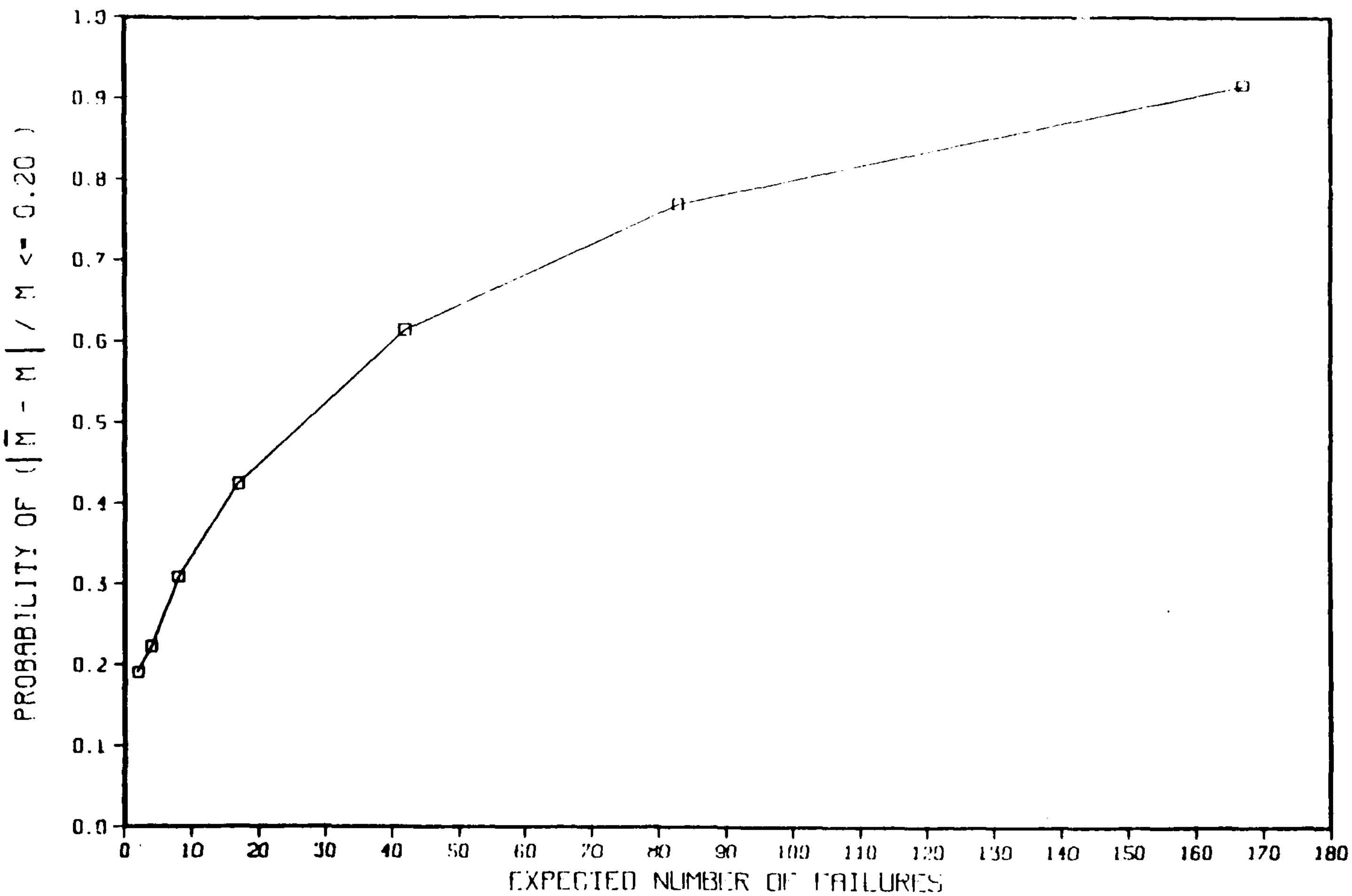


Figure 5. Probability of 20% Precision vs. Expected Number of Failures for $\bar{M}$

rate curves, and then MIL-HDBK-189 estimators $\hat{M}$ and $\overline{M}$ were computed as in Section 2. For both estimators, the distributions of RE were computed. As defined in a previous section, the RE is:

$$RE = |M_{(EST)} - M_{(TRUE)}|/M_{(TRUE)}$$

For the step function construction, $M_{(TRUE)}$ is the MTBF of the system's last configuration. From the simulation results for the discrete failure rate curve, it was observed that the probability of achieving a specified precision strongly depends upon the expected number of failures and weakly upon the number of configurations for about five or more configurations (see Figures 6 and 7). Notice that the $\overline{M}$ estimator behaves in the same way. It is important to note that the probability of achieving a specified precision for a finite number of configurations rapidly approaches the probability obtained for the smooth AMSAA failure rate curve, as the number of configurations increases.

5. APPLICATIONS

The following computational examples will show how to use the relationship between the probability of achieving a specified precision and the expected number of failures to analyze an idealized planning curve.

Computational Examples.

a. Determine the amount of test time to achieve a specified precision with a given probability.

As an example of this type of problem, we shall calculate the amount of test time required to ensure with a probability of 0.80 that the MTBF estimator $\hat{M}$ will be within 20 percent of the true (unknown) MTBF, $M_{(TRUE)}$.

Typically, one attempts to develop an idealized growth curve that will grow to a desired value, M_F. This value, M_F, may be the required MTBF, or it may be a value higher than the required MTBF. The latter case will occur when one is required to demonstrate the desired system's MTBF at a specified confidence level. In either event, we will assume in this example that the end point of the planning curve has been determined and denoted by M_F. Assume we actually grow along the idealized curve to the end point after T hours. Then $M_{(TRUE)}$ will equal M_F. In this example, we wish to calculate a value for T that will ensure with a probability of 0.80 that $|\hat{M} - M_{(TRUE)}| \leq (0.20)\ M_{(TRUE)}$. This value of T depends on the expected number of failures associated with the idealized growth curve. The expected number of failures required to ensure a specified relative error with a probability of 0.80 can be found from a family of "Relative Error vs Expected Number of Failures" curves (see Appendix B).

It can be shown that the expected number of failures, E(F), may be expressed as:

$$E(F) = (1/(1-\alpha))\ (T/M_F)$$

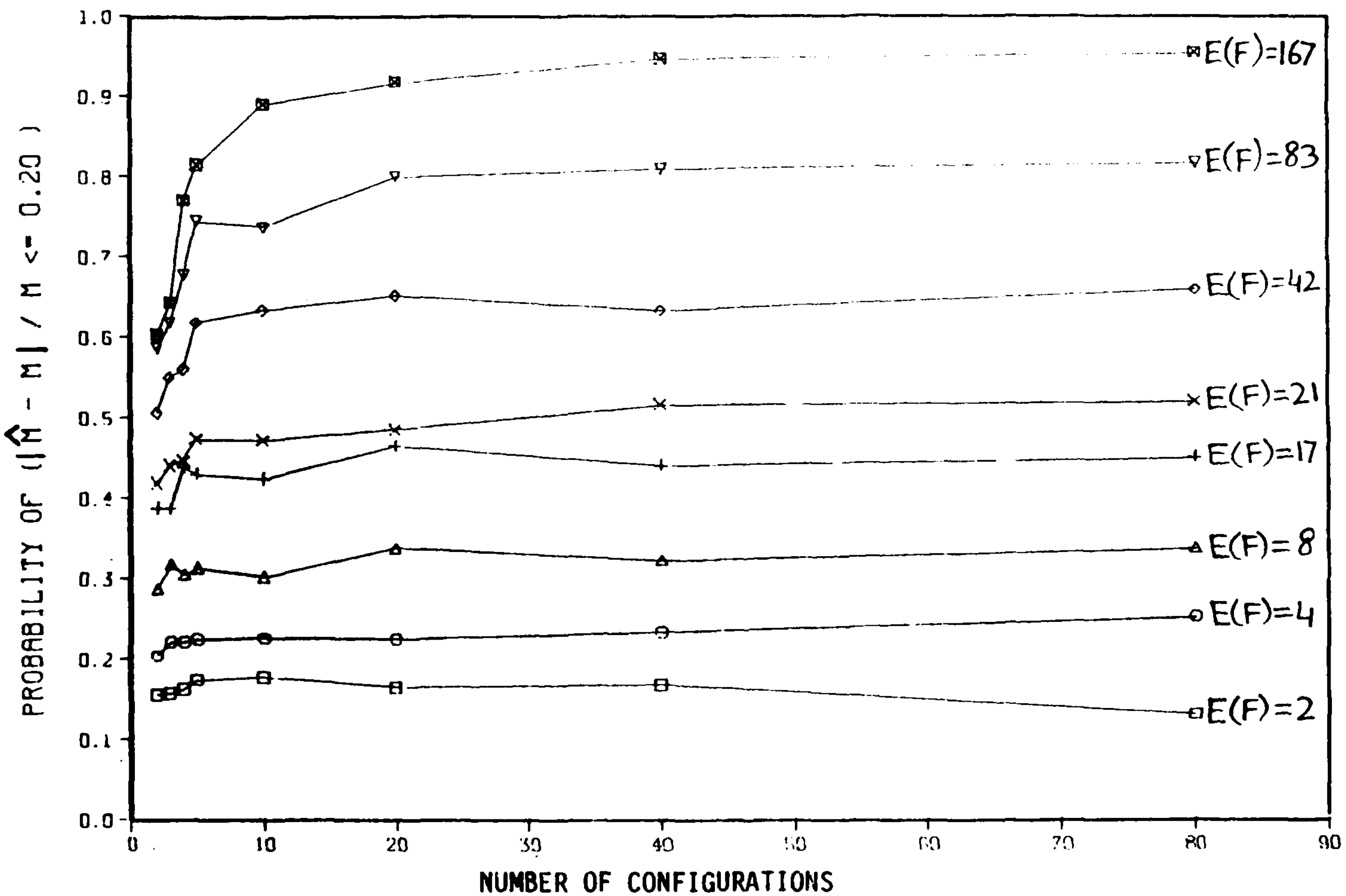


Figure 6. Probability of 20% Precision vs. Number of Configurations for $\hat{M}$

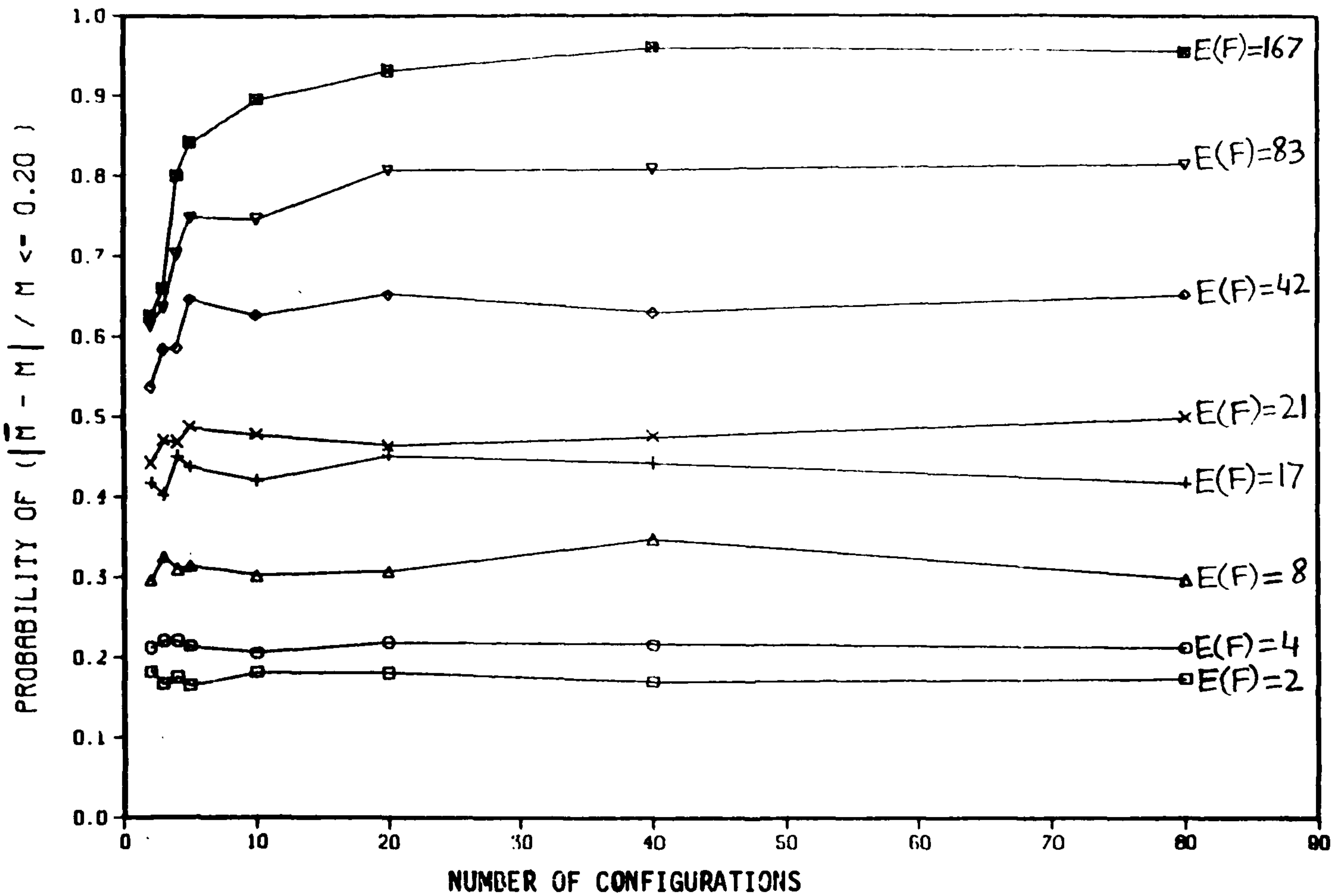


Figure 7. Probability of 20% Precision vs. Number of Configurations for $\overline{M}$

Thus for an assumed growth rate, for example, $\alpha = 0.2$, specified precision $= 0.2$ and $M_F = 168$ hours, we see from Figure 9, Appendix B, that $E(F) = 93$ and T may be calculated as follows:

$$T = (1 - \alpha)(M_F)(E(F))$$

$$= (1 - 0.2)(168)(93)$$

$$= 12499.2 \text{ hours}$$

Notice that for a given M_F and $E(F)$, there is a linear relationship between the test time, T, and the growth rate, α. The idealized growth curve that corresponds to the specified growth rate of 0.2 and that grows to the desired MTBF, $M_F = 168$ hours in an amount of time, $T = 12499.2$ hours can be completely specified by using the following relationship:

$$E(F) = \lambda T^{\beta} \text{ where } \beta = 1 - \alpha$$

Solving for λ, we obtain,

$$\lambda = (E(F))T^{-\beta}$$

The equation of the idealized growth curve is now completely specified.

b. For a given amount of test time T and MTBF value M_F to be achieved at the end of test time T, what precision (i.e., relative error) of the $\hat{M}$ estimator can be ensured with a specified probability for a stated growth rate.

In our example, the given amount of test time will be $T = 3542$ hours and M_F will be taken to be 197 hours. We wish to calculate the precision of the $\hat{M}$ estimator that can be achieved with a probability of 0.80 for a growth curve with growth rate $\alpha = 0.3$.

The expected number of failures, E(F), can be calculated to be 26 by using one of the following formulas.

$$E(F) = \left(1/(1-\alpha)\right)(T/M_F)$$

or

$$E(F) = \lambda T^{\beta} = \lambda T^{(1 - \alpha)}$$

From Figure 9, Appendix B, we see that PROB ($RE < 0.36$) = 0.80.

6. CONCLUSIONS

From Section 3 regarding precision, we conclude the following:

a. The probability of achieving a specified precision increases as the expected number of failures increase.

b. The precision of $\hat{M}$ and $\bar{M}$ is essentially the same.

c. It is important to choose idealized planning curve parameters (T, α, λ) to obtain adequate precision (i.e., RE_S) for the estimator at the desired probability level. In particular, we should state the specified precision (i.e., specified relative error), RE_S, and probability level, PR, where

$$\text{PROB}\left(\frac{|M_{(EST)} - M_{(TRUE)}|}{M_{(TRUE)}} < RE_S \right) = PR$$

d. For a given idealized planning curve and test time, one can calculate the risk that the estimate will not be within a specified percent of the true MTBF, e.g., for the specified percent = 20 percent

$$\text{Risk} = \text{PROB}\left(|M_{(EST)} - M_{(TRUE)}| > 0.20\, M_{(TRUE)} \right)$$

e. This study emphasizes the need to include the confidence bounds on the true MTBF when presenting evaluations based on the MIL-HDBK-189 MTBF estimators. It could be misleading to only present point estimates in cases where the probability of obtaining good precision is low.

From Section 4 regarding robustness, we conclude that:

a. The precision of MIL-HDBK-189 estimators strongly depends on the expected number of failures.

b. The robustness of $\hat{M}$ and $\bar{M}$ is essentially the same.

c. For a small expected number of failures, although the probability of achieving a specified precision is low, it is *robust* with respect to the number of configurations.

d. For a high expected number of failures, the probability is not robust for a small number of configurations. This emphasizes the need for instituting Test, Analyze, and Fix (TAAF) procedures for long duration growth programs when the expected number of failures is high.

The next page is blank.

APPENDIX A

DISTRIBUTION OF RELATIVE ERROR

The next page is blank.

APPENDIX A

DISTRIBUTION OF RELATIVE ERROR

In this appendix, we shall obtain the distribution function of the relative error, RE, where RE is defined as in Section 3. It will be seen that the distribution function is governed by only one scalar parameter, the expected number of failures for the reliability growth test.

For a time terminated growth test of duration T, the AMSAA continuous model assumes the failure process $\{N_t | 0<t<T\}$ is a nonhomogeneous Poisson process with intensity function $\rho(t) = \lambda\beta t^{\beta-1}$ ($\lambda>0$, $\beta>0$). The variate N_t denotes the cumulative number of failures through test time t. The expected value of N_t, denoted by $E(N_t)$, is given by

$$E(N_t) = \int_0^t \rho(s)ds = \lambda t^{\beta} \qquad (1)$$

for $t\geq 0$.

Let $W = \sum_{i=1}^{N_t} \ln(T/f_i)$

where f_i denotes the cumulative test time to the i^{th} failure. The sum is defined to be zero when $N_t=0$. In Reference 2, the distribution of W conditioned on $N_T=n\geq 1$ is derived. This distribution can be used to obtain the distribution of RE.

Let $n\geq 1$ and let W_n denote W conditioned on $N_T=n$. Define f_{W_n} to be the density function of W_n. In Reference 2, it is shown that

$$f_{W_n}(W) = \frac{(\beta W)^{n-1}e^{-\beta W}\,\beta}{(n-1)!} \quad \text{for } W>0$$

$$= 0 \qquad \text{for } W\leq 0.$$

To further consider the distribution of W_n, let G(r,s) denote a gamma random variable with density function

$$g(x) = \frac{1}{s^r\Gamma(r)}\, x^{r-1}e^{-x/s} \quad \text{for } x>0$$

$$= 0 \qquad \text{for } x\leq 0$$

where $r>0$ and $s>0$.

Thus, $W_n \sim G(n,\beta^{-1})$. We may also express W_n in terms of a Chi-Square variate with 2r degrees of freedom, denoted by χ^2_{2r}. To do so, we note

$$G(r,s) = (s/2)\,\chi^2_{2r} \qquad (2)$$

Thus by (2), with r=n and $s=\beta^{-1}$,

$$W_n \sim (1/2\beta)\ \chi^2_{2n} \tag{3}$$

Recall that the maximum likelihood estimate (MLE) for β is given by

$$\hat{\beta} = N_T/\sum_{i=1}^{N_T} \ln(T/f_i) \tag{4}$$

Let $\hat{\beta}_n$ and $\hat{M}_n$ denote the variates $\hat{\beta}$ and MTBF MLE $\hat{M}$ conditioned on $N_T = n \geq 1$, respectively. Then by (4),

$$\hat{\beta}_n = \frac{n}{W_n} \tag{5}$$

Recall $\hat{M} = T/\hat{\beta}N_T$ and thus

$$\hat{M}_n = T/\hat{\beta}_n n \tag{6}$$

Let $M = 1/\rho(T)$, the instantaneous MTBF at the end of the growth program.

Then, by (5),

$$\begin{aligned} \hat{M}_n/M &= (\lambda T^{\beta}/n)\ (\beta/\hat{\beta}_n) \\ &= (\lambda T^{\beta}/n) \left(\frac{\beta W_n}{n}\right) \\ &= \left(\frac{\lambda T^{\beta}}{2n^2}\right)(2\beta W_n) \end{aligned} \tag{7}$$

Using (7) and (3) we obtain

$$\hat{M}_n/M = \left(\frac{\lambda T^{\beta}}{2n^2}\right) \chi^2_{2n} \tag{8}$$

Next, observe that N_T is a Poisson random variable.

By (1), $E(N_T) = \lambda T^{\beta}$. Thus,

$$Pr(N_T = k) = e^{-\lambda T^{\beta}} \frac{(\lambda T^{\beta})^k}{k!} \tag{9}$$

for k = 0,1,2,...

where Pr denotes the Poisson probability function. Also note

$$RE = \left| \frac{\hat{M}-M}{M} \right| = \left| \frac{\hat{M}}{M} - 1 \right| \tag{10}$$

where $\hat{M}$ denotes the MLE for MTBF defined for $n \geq 1$. Thus, the distribution of RE is determined by that of $\hat{M}/M$.

Let F denote the distribution function of $\hat{M}/M$ and let F_n denote the distribution function of $\hat{M}_n/M$ for $n \geq 1$. Then,

$$F(x) = \sum_{n=1}^{\infty} F_n(x) \frac{Pr(N_T=n)}{\{1-Pr(N_T=0)\}} \quad \text{for } x>0 \tag{11}$$

and

$$F(x) = 0 \text{ for } x \leq 0.$$

By equations (8) and (9), we have that $F_n(x)$ and its coefficient in (11) are solely functions of $\lambda T^{\beta} = E(N_T)$ for each positive integer n. Thus by (10), it follows that the distribution of RE is determined by the parameter $E(N_T)$. Furthermore, the distribution function of RE may be evaluated for a given value of λT^{β} via formulas (10), (11), (8), and (9).

The next page is blank.

APPENDIX B

SPECIFIED RELATIVE ERROR VS EXPECTED NUMBER OF FAILURES

The next page is blank.

APPENDIX B
SPECIFIED RELATIVE ERROR VS EXPECTED NUMBER OF FAILURES

To generate the "Specified Relative Error vs Expected Number of Failures" graph, we used the family of "Probability of achieving RE_S vs RE_S" curves (see Figure 8) in which the expected number of failures is constant for a particular curve. These curves depict the variation in the probability of achieving the system's true MTBF within a specified precision for the given expected number of failures.

Most often, we are interested in finding the total test time for a specified precision, say 0.20, and a given probability, say 0.80. Though it is possible to accomplish this objective by directly using the family of curves in Figure 8, it is more convenient to use the graph in Figure 9. To construct this graph, we draw a horizontal line in Figure 8 at a point corresponding to the given probability, say 0.80. From the points of intersection between the line and the curves, draw perpendiculars to the horizontal axis representing the specified relative error (RE_S), see Figure 10. Record the specified relative errors corresponding to the points of intersection between the perpendiculars and the horizontal axis. We know that the expected number of failures remains constant along a particular curve and it varies for different curves. In this way, we can collect data representing the values for the expected number of failures and the corresponding specified relative errors. Then this information is used to construct the desired curve, i.e., Figure 9.

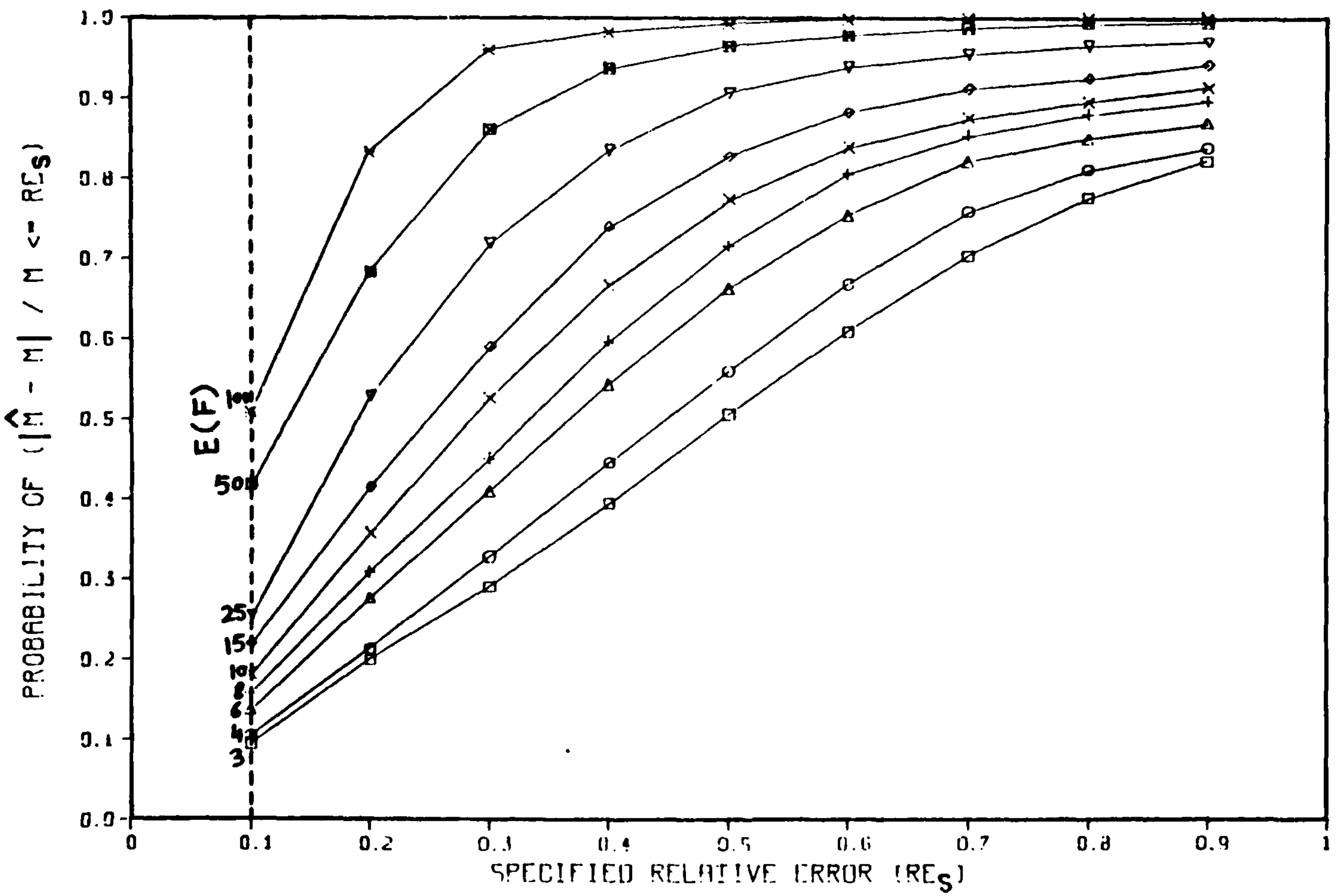


Figure 8. Probability of Achieving RE_S vs RE_S.

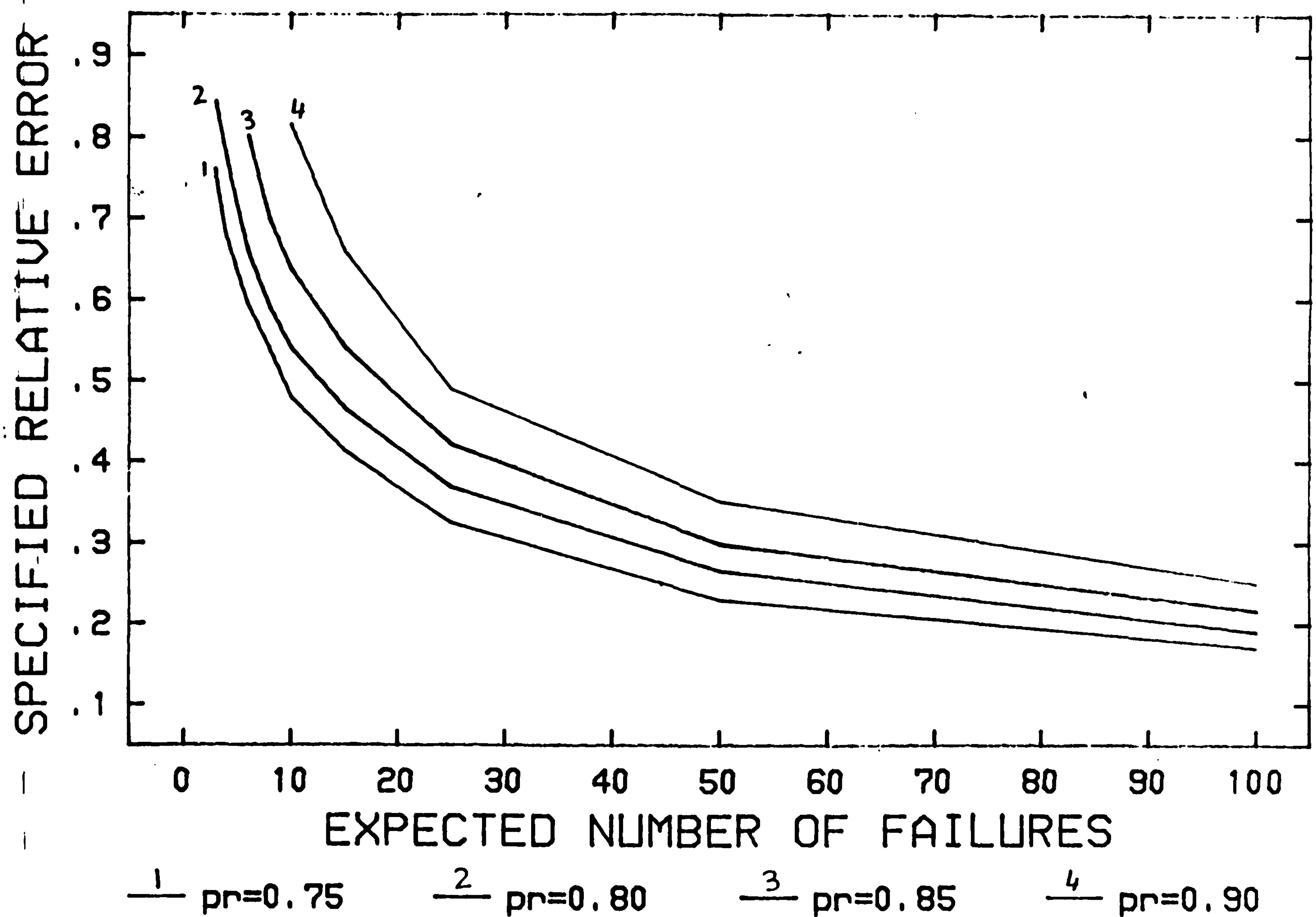


Figure 9. Specified Relative Error vs. Expected Number of Failures

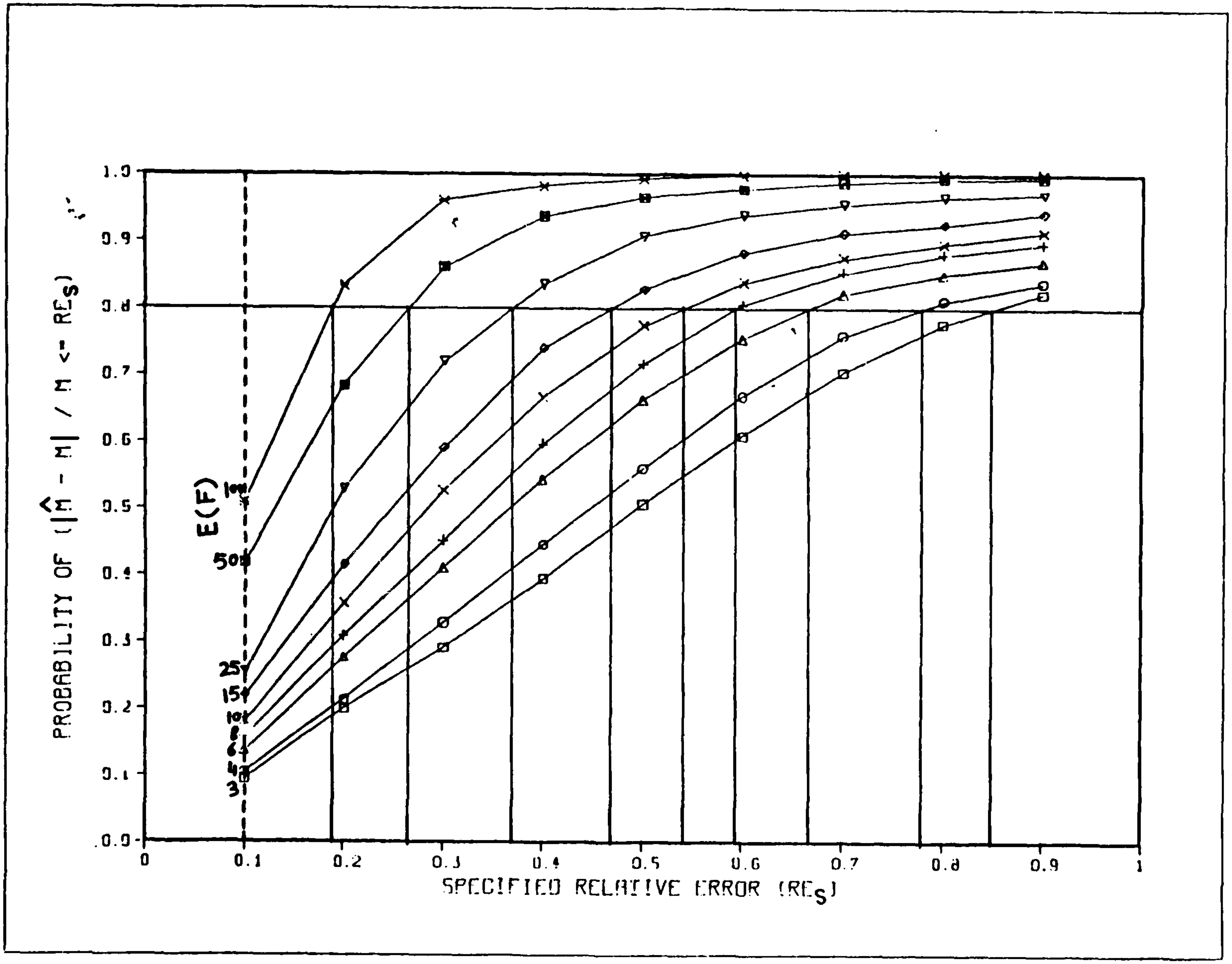


Figure 10. Probability of Achieving RE_S vs RE_S.

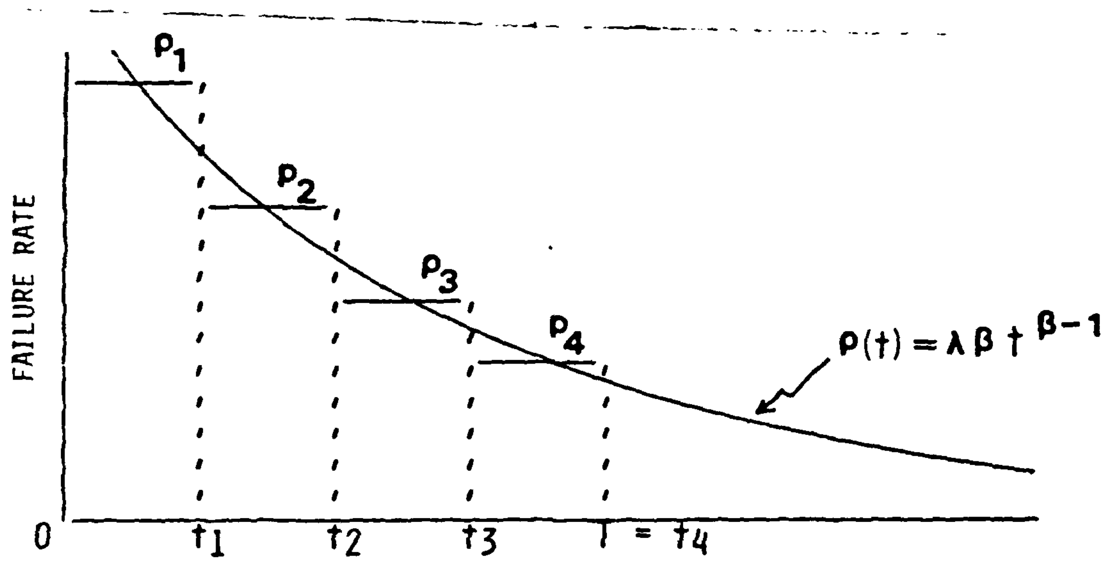
APPENDIX C

CONSTRUCTION OF STEP FUNCTION FAILURE RATE CURVES

The next page is blank.

APPENDIX C
CONSTRUCTION OF STEP FUNCTION FAILURE RATE CURVES

The total test time T is divided into s equal size sub-intervals. For each sub-interval i (i=1,...,s), the constant failure - rate ρ_i is defined as the average value of $\rho(t) = \lambda\beta t^{\beta-1}$ over sub-interval i. This construction is represented graphically for s = 4 in Figure 11.



ρ_i = HEIGHT OF iTH RECTANGLE (i=1,2,3,4)

t_i = i (T/s) = i(T/4)

Figure 11. Step Function Failure Rate Curve.

The next page is blank.

REFERENCES

1. MIL-HDBK-189, "Reliability Growth Management," 13 February 1981.

2. Crow, Larry H., AMSAA TR-197, "Confidence Interval Procedures for Reliability Growth Analysis," June 1977.

The next page is blank.

DISTRIBUTION LIST

No. of Copies	Organization
7	Commander Defense Technical Information Center ATTN: DTIC-FDAC Cameron Station, Bldg 5 Alexandria, VA 22304-6145
8	Commander U.S. Army Materiel Command ATTN: AMCRM AMCMI AMCDE AMCDE-Q AMCQA (Mr. S. Lorber) AMCAE 5001 Eisenhower Avenue Alexandria, VA 22333-0001
2	Director U.S. Army Armament RD&E Center ATTN: SMCAR-TSS Picatinny Arsenal, NJ 07806-5000
1	Commander HQ AMCCOM ATTN: AMSMC-IMP-L Rock Island, IL 61299-6000
2	Director U.S. Army TRADOC Analysis Activity ATTN: ATOR-TSL ATOR-T White Sands Missile Range, NM 88002-5502
1	Commander U.S. Army Missile Command ATTN: AMSMI-OR-SA Redstone Arsenal, AL 35898-5060
1	Commander U.S. Army Troop Support Command ATTN: AMSTR-B 4300 Goodfellow Blvd., St. Louis, MO 63120-1798

No. of Copies	Organization
1	Commander Harry Diamond Lab, Vista Office ATTN: SLCHD-D-PM-V (S. Kovel) 2800 Powder Mill Road Adelphi, MD 20783-1197
2	Commander U.S. Army Tank-Automotive Command ATTN: AMSTA-TSL AMSTA-V Warren, MI 48397-5000
1	Commander U.S. Army Aviation Systems Command Directorate for Plans and Analysis ATTN: AMSAV-BC 4300 Goodfellow Blvd., St. Louis, MO 63120-1798
1	Commander U.S. Army Natick RD&E Center ATTN: STRNC-O Natick, MA 0176--5000
2	Commander U.S. Army Concepts Analysis Agency ATTN: CSCA-MSI-L CSCA-MV-M 8120 Woodmont Avenue Bethesda, MD 20814-2797
1	Commandant U.S. Army Infantry School ATTN: ATSH-CD-CS-OR Fort Benning, GA 31905-5400
1	Pentagon Library ATTN: ANR-AL-RS (Army Studies) Pentagon, Rm 1A518 Washington, DC 20310

DISTRIBUTION LIST (Continued)

No. of Copies	Organization	No. of Copies	Organization
3	Chief Defense Logistics Studies Information Exchange U.S. Army Logistics Management Center ATTN: AMXCM-D (2 copies) AMXCM-ACM (Mr. Fowler) Fort Lee, VA 23801-6043	1	Commander USAREUR & 7th Army ATTN: AEAG-X-OR APO New York 09403
1	Director Combat Data Information Center AFWAL/FIES/CDIC Wright-Patterson AFB, OH 45433-5000		

Aberdeen Proving Ground

No. of Copies	Organization	No. of Copies	Organization
2	Cdr, USATECOM ATTN: AMSTE-AD AMSTE-TO-F AMSTE-CL (MAJ Stewart) Ryan Bldg APG, MD M21005-5055	1	Dir, BRL ATTN: SLCBR-OD-ST Bldg: ~~305~~ 328 APG, MD 21005-5066
1	Dir, BRL, Bldg: 328 ATTN: SLCBR-D APG, MD 21005-5066	1	Dir, HEL ATTN: SLCHE-FS (Dr. M. Hofmann) Bldg: 459 APG, MD 21005-5001
4	Dir, AMSAA ATTN: AMXSY-R (T. Ziad) AMXSY-PA (3 cys)		

GIST

US ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY AMSAA	TITLE Statistical Precision and Robustness of the AMSAA Continuous Reliability Growth Estimators BRIEFING ____ REPORT TR 453	

THE PRINCIPAL FINDINGS

1. The probability of achieving a specified precision solely depends upon the expected number of failures when the failure data are generated from the AMSAA Continuous failure rate curve.

2. The probability of achieving a specified precision still strongly depends upon the expected number of failures when failure data come from a finite number of configurations.

3. Precision and Robustness of MTBF estimators $\hat{M}$ and $\overline{M}$ are essentially the same.

THE MAIN ASSUMPTIONS

1. For the precision part of the study, the failure data are assumed to be generated from the AMSAA continuous failure rate curve.

2. For the robustness part of the study, the failure data are assumed to be generated from a finite number of configurations whose overall trend follows the AMSAA failure rate curve.

3. We studied robustness with respect to the number of configurations by choosing equal configuration time periods.

THE PRINCIPAL LIMITATIONS We used (MIL-HDBK-189) MTBF estimators' formulas under the assumption that failure data are coming from the AMSAA continuous failure rate curve for the precision part. We used the same formulas to study robustness with the assumption that failure data are coming from a finite number of equal configuration time periods.

THE SCOPE OF THE STUDY

This methodology can be used to calculate the required test time associated with an idealized planning curve to achieve a specified precision with a given probability.

THE STUDY OBJECTIVE

1. To study the statistical precision of the AMSAA (MIL-HDBK-189) MTBF estimators $\hat{M}$ and $\overline{M}$.

2. To study robustness, i.e., the effect on the estimator statistical precision due to the discrete configuration changes.

THE BASIC APPROACH

1. To determine the precision of the MIL-HDBK-189 MTBF estimators, we did 5000 simulations to generate a distribution of relative error of the MTBF estimators. The failure data were generated from the AMSAA continuous failure rate curve.

2. To determine the robustness, we generated a distribution of relative error from a finite number of configurations through 5000 simulations.

REASON FOR PERFORMING THE STUDY

The study results are useful for planning purposes. We can determine the test time for a probability and a specified precision level.

IMPACT

Reliability growth plan.

STUDY SPONSOR

U.S. Army Materiel Systems Analysis Activity (USAMSAA), RAM Division

PRINCIPAL INVESTIGATORS

Tariq Ziad and Paul Ellner